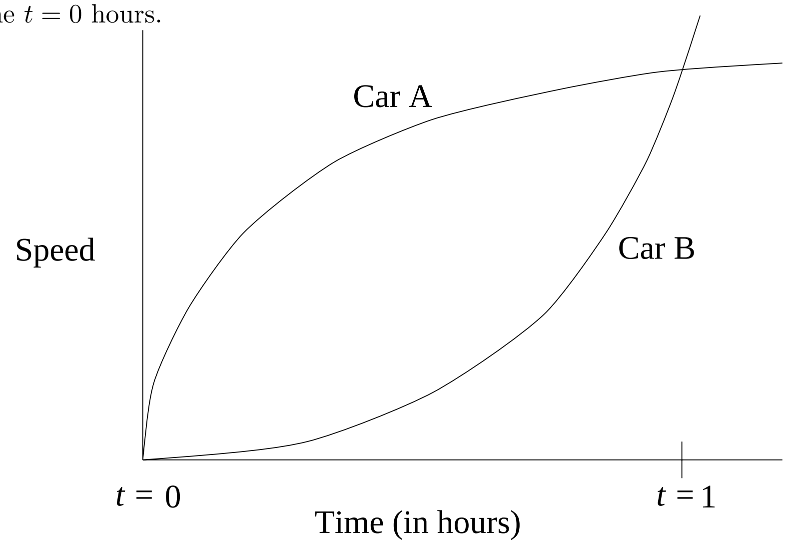
WORKSHEET 3 - Fall 1995

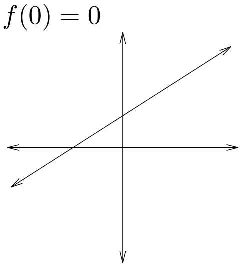
1. The following graph represents speed vs. time for two cars, A and B. Assume the cars are at the same position at time $t = 0$ hours.



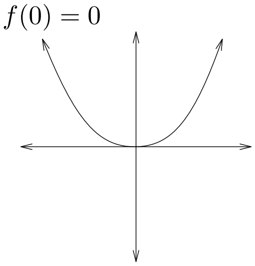
   a) State the relationship between the position of car A and car B at $t = 1$ hour.
   b) State the relationship between the speed of car A and car B at $t = 1$ hour.
   c) State the relationship between the acceleration of car A and car B at $t = 1$ hour. Explain.
   d) What is the relative position of the two cars during the time interval between $t = 0.75$ hours and $t = 1$ hour? (e.g. is one car pulling away from the other?) Explain.

2. In each part below, you are given the graph of the derivative f' of some function f. Sketch a graph of f satisfying the condition listed.

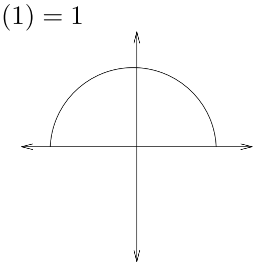
   a) $f(0) = 0$



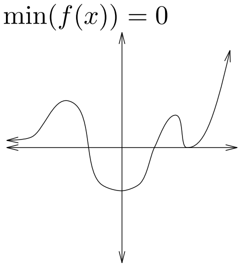
   b) $f(0) = 0$



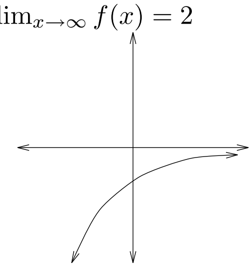
   c) $f(1) = 1$



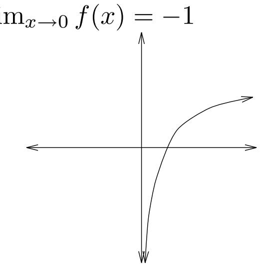
   d) $\min(f(x)) = 0$



   e) $\lim_{x\to\infty} f(x) = 2$



   f) $\lim_{x\to 0} f(x) = -1$

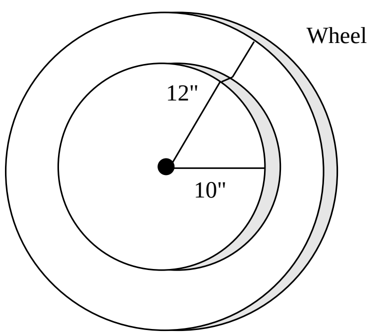


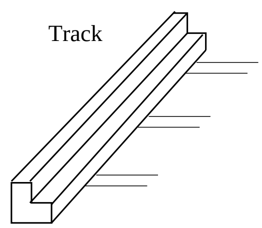
3. While studying calculus late one night in his dorm room, Bob decides to take a study break and hone his yo-yo skills. He soon realizes that there is interesting mathematics in the motion of his favorite toy (Bob has been studying calculus too long.)
   a) Sketch a graph of the velocity of a bouncing yo-yo as a function of time.
   b) Sketch a graph of the position of the yo-yo as a function of time.
   c) Does your answer to part a) look like the derivative of your answer to part b)?

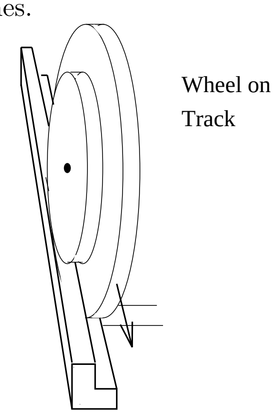
4. Recall that *Newton's method* is an algorithm for estimating zeros of a function. The process begins with an initial guess x_0. If $f(x_0) \neq 0$ then we obtain a refinement of this guess. This is done by approximating f with its tangent line at the point $(x_0, f(x_0))$ and finding the x-intercept of this line. This value is our next guess x_1. This process is repeated until we obtain an estimate which meets our satisfaction.
   a) Apply Newton's method to the function $f(x) = x^{1/3}$ and initial guess $x_0 = 1$. Compute x_1 and x_2. (You need to know that $f'(x) = \frac{1}{3}x^{-2/3}$.) What is happening here?
   b) Perhaps setting $x_0 = 1$ was simply not clever enough and choosing an initial guess closer to zero would work better. Prove or disprove this.
   c) Suppose the quadratic $ax^2 + bx + c$ has two real roots (i.e. two zeros) and that you are trying to determine the larger of the two. Given an initial guess x_0, how could you decide whether Newton's method would converge to the larger or smaller root. (Recall that the derivative of this quadratic is $2ax + b$.)

5. Imagine a road on which the speed limit is specified at every single point. In other words, there is a certain function L such that the speed limit x miles from the beginning of the road is $L(x)$. Two cars, A and B, are driving along this road; car A's position at time t is $a(t)$, and car B's is $b(t)$.
   a) What equation expresses the fact that the car A always travels at the speed limit? (Hint: the answer is not $a'(t) = L(t)$.)
   b) Suppose that A always goes at the speed limit, and that B's position at time t is A's position at time $t - 1$. Show that B is also going at the speed limit at all times.
   c) Suppose B always stays at a constant distance behind A. Under what circumstances will B still always travel at the speed limit?

6. A train wheel has two rims as shown below and is designed to run along a bi-level track. The outer (smaller) rim has radius 10 inches while the inner (larger) rim has radius 12 inches.







   a) How far does the outer rim roll over the course of one rotation of the wheel?
   b) How far does the inner rim roll over the course of one rotation of the wheel?
   c) How far does the train travel over the course of one rotation of the wheel?

7. A bear walks one mile due south, then one mile due east, and then one mile due north. He ends up at exactly the same spot where he started. What color is the bear? Give a rigorous proof.