CHEBYSHEV COVERS AND EXCEPTIONAL NUMBER FIELDS

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Abstract. Among the simplest of the classical polynomials are the Chebyshev polynomials of the first and second kind, $T_k(x)$ and $U_k(x)$. In our normalization, the indices are allowed to be half- integers as well as integers, and the "polynomials" actually live in $\mathbb{Z}[x, \sqrt{2-x}, \sqrt{2+x}]$.

We will show that the rational functions $\dfrac{T_{m/2}(x)^n}{T_{n/2}(x)^m}$ and $\dfrac{U_{m/2}(x)^{2n}}{U_{n/2}(x)^{2m}}$ are very remarkable from the point of view of Grothendieck's *dessins d'enfants*. The fibers of these rational functions are likewise very remarkable from the point of view of algebraic number theory. For example, for $(m, n) = (125, 128)$ the fiber of the second function above 5 is given by a degree 15875 polynomial in $\mathbb{Z}[x]$ with discriminant $-2^{130729}5^{63437}$ and Galois group the entire symmetric group S_{15875}.