Math 300
Supplementary Discussion

Hasse Diagrams of Posets

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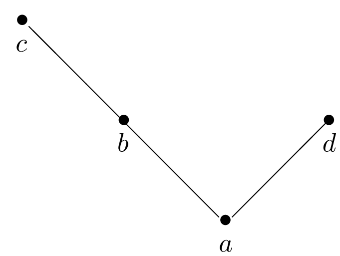
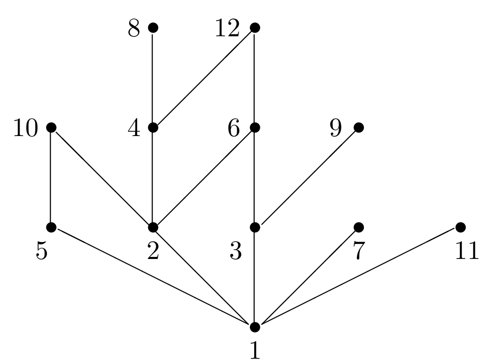
A finite partially ordered set can be represented by a "Hasse diagram" as follows.

Let A be a finite set, and suppose R is a partial ordering of A. Each element of A corresponds to a vertex of the Hasse diagram for the poset (A, R). Moreover, if $(a, b) \in R$, then in the Hasse diagram, the vertex corresponding to a is lower than the vertex corresponding to b, *and* there is an ascending path in the Hasse diagram from vertex a to vertex b. WARNING: Hasse diagrams are only used for finite partially ordered sets!

Example 1. Let $A = \{a, b, c, d\}$ and let $R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, c), (a, c), (a, d)\}$. Then R is a partial ordering of A. The Hasse diagram for the poset (A, R) is shown below.



Example 2. Let $A = \{1, 2, 3, \ldots, 12\}$ and let $D = \{(x, y) \in A \times A : x \mid y\}$. Then D is a partial ordering of A. The Hasse diagram for the poset (A, D) is shown below.

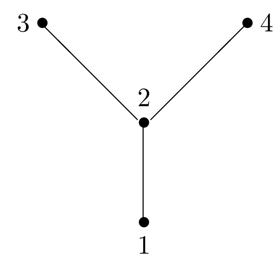


Let A be a finite set, and let R be a partial ordering of A. Suppose you have the directed graph of the relation R. How do you construct the Hasse diagram for (A, R)?

1. Remove all the loops.
2. Arrange the vertices so that all directed edges point upward.
3. Remove any edge whose presence is implied by the transitive property.
4. Remove the arrowheads from the edges.

Warning: since Hasse diagrams are only used for partially ordered sets, before you begin the above procedure be certain that your directed graph represents a partial ordering.

Example 3. The Hasse diagram obtained from the directed graph in Exercise 4.4.3 of Velleman is shown below.



Example 4. The Hasse diagram obtained from the directed graph in Figure 3, Section 4.3 of Velleman is shown below.