

Lecture 4, Th., Jan., 25, 2007

Reading homework: Kozusko and Bajzer (2003), Combining Gompertzian growth and cell population dynamics, Math. Biosc. 185, 153-167

We covered the second part of the above paper (i.e. [9]).

1. Nutrient Limited Growth as a Plausible Mechanism for Gompertz Model. Recall that the original 1941 von Bertalanffy model is formulated with the assumption that a multicellular tumor spheroid (MCS) has a proliferation term proportional to the spheroid surface area (since nutrient enters the tumor through surface diffusion) and its death rate is a constant. The constant death rate for all the cells in a tumor is clearly a bit stretch from reality as the nutrient is concentrated near the surface where cells proliferate and suffer little or no death, while cells far away from surface are likely die due to starvation. To capture this fact, we assume below the cell death rate is proportional to the cell distance from the spheroid surface. In other words, we assume that the death rate $d(r) = mr$ where r is the cell distance from the spheroid surface.

Let V be the tumor (MCS) volume and assume that the mass density is constant, say equal to 1. Then the radius of the tumor is $R \equiv cV^{1/3} \equiv (3/(4\pi))^{1/3}$. Hence, we have

$$(1.1) \quad V' = \alpha V^{2/3} - \int_0^R 4\pi m(R-r)r^2 dr = \alpha V^{2/3} - \frac{m\pi}{3} V^{4/3} \equiv B(V).$$

Figure 1.1 shows that the von Bertalanffy growth function $1.475(x^{2/3} - x^{4/3})$ is very close to Gompertz growth function $-x \ln(x)$ qualitatively and quantitatively. Indeed, it even provides a slightly better fit than the Gompertz growth function for the data depicted in Fig. 2.2 of Lecture 2.

Matlab file for generating Figure 1.1.

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x = 0:0.005:1;  
y1 = x.*(1-x);  
y2 = 1.475*(x.^(2/3)-x.^(4/3));  
y3 = -x.*log(x);  
plot(x,y1, 'r:', x, y2, x, y3,'k-', 'linewidth',2)  
legend('logistic','von Bertalanffy','Gompertz',1)  
ylim([0 0.4]);  
xlabel('N')  
ylabel('tumor growth functions')  
title('Plot of some tumor growth functions','FontSize',12)  
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```

EXERCISES

Exerciser (1)-(4) are DUE on TH., Feb. 1

(4). Assume that a multicellular tumor spheroid has a proliferation term proportional to the spheroid surface area and a death rate is a function $d(r)$ where r is the cell distance from the spheroid surface. Derive such a tumor growth model if $d(r) = m(e^{ar} - 1)$.

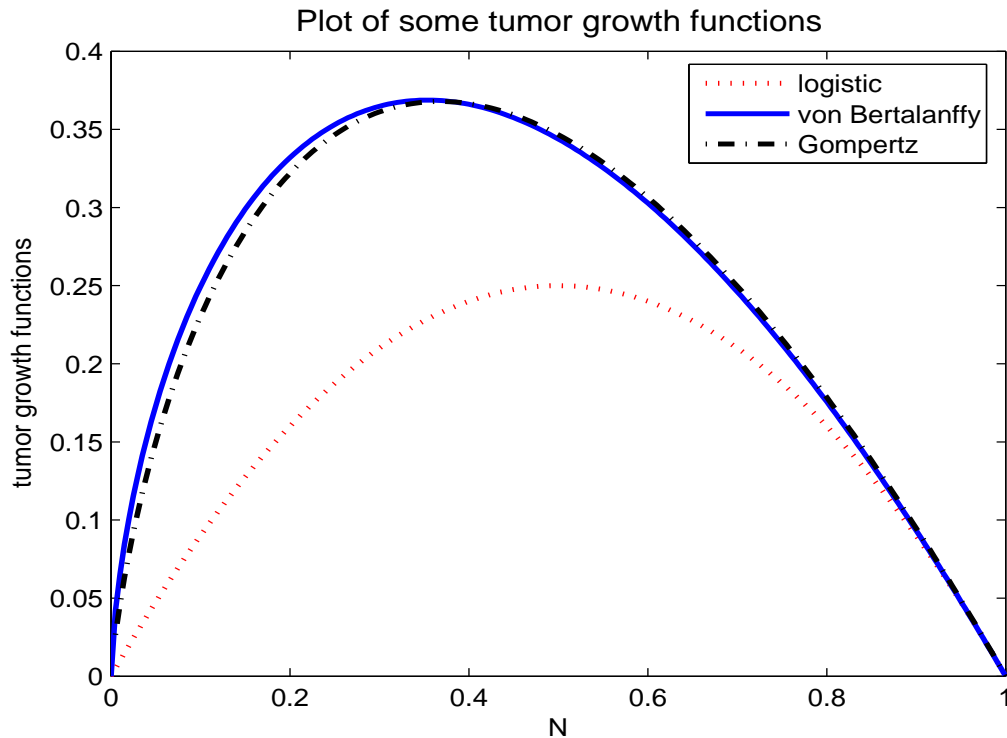


FIG. 1.1. This figure shows that the von Bertalanffy growth function $1.475(x^{2/3} - x^{4/3})$ is very close to Gompertz growth function $-x \ln(x)$. Indeed, it even provides a slightly better fit than the Gompertz growth function for the data depicted in Fig. 2.2 of Lecture 2.

REFERENCES

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