Lecture 3, T., Jan., 23, 2007

Reading homework: Kozusko and Bajzer (2003), Combining Gompertzian growth and cell population dynamics, Math. Biosc. 185, 153-167

We covered the first four section of the above paper (i.e. [1]).

1. A Plausible Mechanism for Gompertz Model. In [1], a two-compartment model of cancer cells population dynamics proposed by Gyllenberg and Webb includes transition rates between proliferating and quiescent cells as non-specified functions of the total population, N is introduced. The net inter-compartmental transition rate function is assumed to be a function of the total cells. This net inter-compartmental transition rate function can be selected to generate the Gompertz growth model. Effectively, this leads to a hybrid model for which explicit analytical solutions for proliferating and quiescent cell populations, and the relations among model parameters can be obtained. The model predicts that the number of proliferating cells may increase along with the total number of cells, but the proliferating fraction appears to be a continuously decreasing function. The net transition rate of cells is shown to retain direction from the proliferating into the quiescent compartment. The death rate parameter for quiescent cell population is shown to be a factor in determining the proliferation level for a particular Gompertz growth curve.

Matlab file for generating Figure 1.1 containing some curves of panels (a), (b) and (c) in Figure 3 in [1] with end time $T = 80$.

```
-------------------------------------------------------------------
x = 0:0.1:80;
b=2.76; a=0.134; uq=0.05;
y = exp((b/a)*(1-exp(-a*x)))/10^8;
y1 = 10*y.*(uq+b*exp(-a*x))/(uq+b);
y2= (uq+b*exp(-a*x))/(uq+b);

subplot(131),
plot(x,y,'r-.', 'linewidth',2)
legend('Gompertz curve', 2)
ylim([0 9]); xlim([0 80]);
text(28,4,'b=2.76, a=0.134','FontSize',12)
text(5,8,'(a)','FontSize',12)
xlabel('t(y)')
ylabel('N(t) (x 10^8)')
title('Figure 3(a)','FontSize',12)

subplot(132),
plot(x,y1,'b', 'linewidth',2)
legend('P(t)/P_0', 2)
ylim([0 6]); xlim([0 80]);
text(45,2,'\mu_q=0.05','FontSize',12)
text(5,5,'(b)','FontSize',12)
```

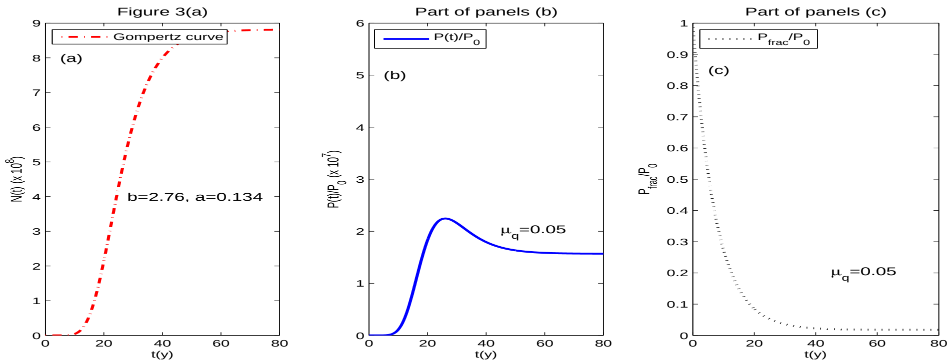

FIG. 1.1. *This figure contains some curves of panels (a), (b) and (c) in Figure 3 in [1] with end time $T = 80$.*

```
xlabel('t(y)')
ylabel('P(t)/P_0 (x 10^7)')
title('Part of panels (b)','FontSize',12)

subplot(133),
plot(x,y2,'k:', 'linewidth',2)
legend('P_{frac}/P_0', 2)
ylim([0 1]); xlim([0 80]);
text(5, 0.85,'(c)','FontSize',12)
text(45,0.2,'\mu_q=0.05','FontSize',12)
xlabel('t(y)')
ylabel('P_{frac}/P_0')
title('Part of panels (c)','FontSize',12)
-------------------------------------------------------------------
```

EXERCISES

(2). Find the expression of $\phi(N)$ so that the total population $N = P + Q$ growth in the Gyllenberg-Webb model

$$P' = (\beta - \mu_p)P - \phi(N) \tag{1.1}$$

$$N' = (\beta - \mu_p + \mu_q)P - \mu_q N \tag{1.2}$$

is equivalent to the logistic growth $N' = rN(1 - N/K)$.

(3). Reproduce Figure 3 in [1] with end time $T = 80$ instead of 60.

REFERENCES

[1] F. Kozusko and Z. Bajzer (2003), *Combining Gompertzian growth and cell population dynamics*, Math. Biosc. 185, 153-167

[2] J. A. Adam and N. Bellomo (1997), *A Survey of Models for Tumor-Immune System Dynamics*, Birkhauser,Boston.

[3] A.M. Parfitt and D.P. Fyhrie (1997), *Gompertzian growth curves in parathyroid tumors: further evidence for the set-point hypothesis*, Cell Proliferat., **30**, 341-349.