

Concepts and Formulas for Tests

Coordinate plane

- Distance between points: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$;
- Midpoint: $x = \frac{x_1 + x_2}{2}$; $y = \frac{y_1 + y_2}{2}$.
- Intercepts:
 - x -intercept: $y = 0$;
 - y -intercept: $x = 0$.
- Symmetry:
 - y -axis symmetry: $x \rightarrow (-x)$;
 - x -axis symmetry: $y \rightarrow (-y)$;
 - origin symmetry: $x \rightarrow (-x)$ and $y \rightarrow (-y)$.
- Circle: $(x - a)^2 + (y - b)^2 = r^2$.
- Straight lines:
 - slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$;
 - Point-slope equation: $y = y_1 + m(x - x_1)$;
 - Slope-intercept equation: $y = mx + b$;
 - General equation: $Ax + By = C$.
 - Parallel lines: $m_2 = m_1$.
 - Perpendicular lines: $m_2 = -\frac{1}{m_1}$.

Equations and inequalities

- Word problems.
- Quadratic equations: $ax^2 + bx + c = 0$.
 - factoring: $x^2 + bx + c = (x \pm \dots)(x \pm \dots)$
 - completing square: $x^2 + bx = -c$;
$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$
$$\left(x + \frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$
 - quadratic formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

- Complex numbers: $a + bi$; $i = \sqrt{-1}$; $i^2 = -1$.

- addition/subtraction — as usual;
- multiplication — use FOIL rule:

$$(a + bi)(c + di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$$

- division: $\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{\dots}{c^2 + d^2}$

- square roots of negative numbers: $\sqrt{-c} = (\sqrt{c})i$

- Inequalities:

1. Bring everything to one side (the other side is 0);
2. Replace the inequality with “=” and find solutions.
If there is a denominator, set it equal to 0 and find prohibited values.
3. Draw the real line with all solutions and prohibited values. Identify intervals (“pieces”) and sample values.
4. Substitute all sample values into the inequality.
5. Identify “correct” intervals and write down the answer. Do not include any prohibited values!

- Absolute value:

- equation $|A| = b$ means $A = b$ or $A = -b$;
- inequality $|A| < b$ means $-b < A < b$;
- inequality $|A| \leq b$ means $-b \leq A \leq b$;
- inequality $|A| > b$ means $A < -b$ or $A > b$
- inequality $|A| \geq b$ means $A \leq -b$ or $A \geq b$

Functions

- $f(x) =$ something with x .
- Given an equation with x and y . Is it a function?
 - Solve for y
 - If solution is unique — it’s a function.
 - If solution isn’t unique (\pm etc.) — it’s not a function.
- Piecewise functions: check x and pick correct formula.
- Domain: values of x for which $f(x)$ makes sense.
 - something with \sqrt{A} means $A \geq 0$.
 - something with $\frac{\dots}{A}$ means: $A = 0$ makes prohibited values: find and exclude them.

- Graph of a function
- Vertical line test:
 - if every vertical line has one intersection — it's a function;
 - if 2 or more intersections — it's not a function.
- Even function: $f(-x) = f(x)$
- Odd function: $f(-x) = -f(x)$
- Intervals of increase and decrease — take into account x -values of points
- Transformations of graphs:
 - function $f(x) + c$ means c units up \uparrow
 - function $f(x) - c$ means c units down \downarrow
 - function $f(x + c)$ means c units to the left \leftarrow
 - function $f(x - c)$ means c units to the right \rightarrow
 - function $-f(x)$ means reflection through x -axis
 - function $f(-x)$ means reflection through y -axis
 - function $cf(x)$ means vertical stretch (for $c > 1$) or shrink (for $c < 1$)
- Combinations of functions:
 - sum: $(f + g)(x) = f(x) + g(x)$
 - difference: $(f - g)(x) = f(x) - g(x)$
 - product: $(fg)(x) = f(x)g(x)$
 - quotient: $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$
 - composition: $f \circ g(x) = f(g(x))$. Solved from inside out.
- Inverse function $f^{-1}(x)$. $f(a) = b$ means $f^{-1}(b) = a$.
- Given $f(x) = \dots x \dots$ How to find $f^{-1}(x)$?
 - set $y = \dots x \dots$
 - put x for y and y for x . Obtain $x = \dots y \dots$
 - solve for y . The result is $f^{-1}(x)$.
 - If solution is unique — there is inverse $f^{-1}(x)$
 - If solution isn't unique (\pm etc.) — there is no inverse.
- Graph of $f^{-1}(x)$: reflection of graph of $f(x)$ through line $y = x$ (at 45 degrees).
- Horizontal line test
 - if every horizontal line has one intersection — it's a function;
 - if 2 or more intersections — it's not a function.

Polynomials

- $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$
- End behavior:
 - n even, $a_n > 0$ — both up;
 - n even, $a_n < 0$ — both down;
 - n odd, $a_n < 0$ — down on left, up on right;
 - n odd, $a_n > 0$ — up on left, down on right.
- Division of polynomials:
 - Long division;
 - Synthetic division.
- Remainder theorem: when dividing $p(x)$ by $(x - c)$, the remainder equals $p(c)$.
- Zeros of polynomials
- Rational zeros: $\pm \frac{p}{q}$ where p is a factor of a_0 , and q is a factor of a_n .
- On a calculator: use TABLE. Start= $-a_0$, End= a_0 , pitch= $1/a_n$.
- Complex zeros: if $a + bi$ is a zero, then $a - bi$ is also a zero.
- If c_1, c_2, \dots, c_n are zeros of $p(x)$, then $p(x) = a(x - c_1)(x - c_2) \cdots (x - c_n)$.

Rational functions

- $f(x) = \frac{\text{polynomial}}{\text{polynomial}}$
- Vertical asymptotes — same as prohibited values: denominator = 0
- (# pieces) = (# vertical asymptotes) + 1
- Horizontal asymptotes: $f(x) = \frac{a_n x^n + \cdots}{b_m x^m + \cdots}$
 - if $n < m$: horiz. asym. is the x -axis (equation $y = 0$)
 - if $n > m$: none
 - if $n = m$: horiz. asym. is $y = \frac{a_n}{a_m}$

Exponents and logarithms

- Exponential function: $f(x) = a^x$, where $a > 0$.
- Graph:
 - y -intercept: $(0,1)$
 - x -intercept: none
 - horizontal asymptote: x -axis
 - for $a > 1$ — increasing, for $0 < a < 1$ — decreasing
- Properties of exponents:

$$\begin{aligned}a^{b+c} &= a^b \cdot a^c \\ a^{b-c} &= \frac{a^b}{a^c} \\ (a^b)^c &= a^{bc}\end{aligned}$$

- **Logarithm:** $\log_a b = c$ if $a^c = b$
- Logarithmic function: $f(x) = \log_a x$ where $a > 0$ and $a \neq 1$.
- Domain: $x > 0$; $(0, \infty)$.
- Graph:
 - x -intercept: $(1,0)$
 - y -intercept: none
 - vertical asymptote: y -axis
 - for $a > 1$ — increasing, for $0 < a < 1$ — decreasing
 - function $f(x) = \log_a x$ is inverse to function $g(x) = a^x$
- Properties of logarithms:

$$\begin{aligned}\log_a(b \cdot c) &= \log_a b + \log_a c \\ \log_a \frac{b}{c} &= \log_a b - \log_a c \\ \log_a(b^c) &= c \cdot \log_a b \\ \log_a b &= \frac{\log_c b}{\log_c a}\end{aligned}$$

Logarithmic and exponential equations

- Population growth: $P = P_0 e^{rt}$
- Compound interest: $A = P \left(1 + \frac{r}{t}\right)^{nt}$
- Continuous compounding: $A = P e^{rt}$

- Radioactive decay: $M = m_0 e^{-rt}$
- Solving exponential and logarithmic equations:
 1. Isolate the exponent or logarithm;
 2. Eliminate logarithm by applying exponent with the same base;
 3. Eliminate exponent by applying logarithm (\ln);
 4. Solve the equation.

Systems of equations

- Two equations with two unknowns
- Method of **substitution**:
 1. solve one equation for y (isolate y);
 2. plug into the second equation - now it involves x only;
 3. solve it for x ;
 4. plug x into the first equation and solve for y
- Method of **elimination**:
 1. combine equations to eliminate one unknown;
 2. solve for the remaining unknown;
 3. plug into one of equations and solve.