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You need to remember following formulas for test 1:

1. $(x-a)^2+(y-b)^2+(z-c)^2=d^2$ is a sphere center at (*a, b, c*) and radius *d*.
2. $(x-a)^2+(y-b)^2=c^2$ is a circle in 2D center at (*a, b*) and radius *c*
3. $y=x^2$, $x=y^2$ are parabolas
4. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is an ellipsoid center at (0, 0, 0). When $a = b = c$ the graph is a sphere
5. To find dot and cross product of two given vectors
6. Area of a parallelogram in terms of cross product
7. Volume of a parallelepiped in terms of scalar triple product
8. Vector Projection of a vector onto another vector and scalar projection of a vector onto another vector
9. Orthogonal projection of a vector ***b*** onto a vector ***a***: $Orth_a b = b - proj_a b$
10. Unit tangent, unit normal and binormal vectors
11. Writing equation of a line in vector form, symmetric form and parametric form

Review the given problems along with your lecture note and homework

1. Find the center and radius of the sphere $4x^2+4y^2+4z^2-8x+16y-1=0$.
2. Find the center and radius of the sphere $x^2+y^2+z^2=x+y+z$.
3. Find the center and radius of the sphere $x^2+y^2+z^2=4x-2y$.
4. Given a vector $r=<-4,0,-3>$. Find a vector of magnitude 5 in the direction of the given vector.
5. Find the angle between the vectors $u=<2,10,5>$ and $v=<-3,4,5>$
6. Find the sine of the angle between the vectors $u=<2,10,5>$ and $v=<-3,4,5>$
7. Find the scalar and vector projection of the vector $a=<-2,3,1>$ onto $b=<1,1,2>$
8. Find orthogonal projection of $b=<-5,-6,-6>$ onto $a=<1,-4,8>$
9. Find the area of the parallelogram with vertices P(0, 0, 0), Q(-5, 1, 3), R(-5, 0, 1) and S(-10, 1, 4).
10. Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS where P(0, 1, 2), Q(2, 4, 5), R(-1, 0, 1), and S(6,-1, 4)
11. Find a vector equation for the line through the point P(5, 0, -1) and parallel to the vector $v=<-2,3,3>$
12. Find the point of intersection between the lines $\frac{x-14}{6}=\frac{y+1}{3}=\frac{z-24}{7}$ and

    $x+18=\frac{y+10}{3}=\frac{z+17}{10}$

Hint: $\frac{x-14}{6}=\frac{y+1}{3}=\frac{z-24}{7}=t$ and $x+18=\frac{y+10}{3}=\frac{z+17}{10}=s$, then we have the following: $x=6t+14, y=3t-1, z=7t+24$ and $x=s-18, y=3s-10, z=10s-17$

At the intersection $6t+14=s-18, 3t-1=3s-10, 7t+24=10s-17$ and solving for *t* and *s*, $s=6t+32, t=-87/15, s=-14/5$. And we then have from first line $x=-104/5, y=-92/5, z=-83/5$, and from second line $x=-104/5, y=-92/5, z=-45$, which are not the common points. Thus the lines do not intersect, they are skew lines.

13. Find the point of intersection between the lines $\frac{x-14}{6}=\frac{y-1}{2}=\frac{z-24}{7}$ and $\frac{x+18}{12}=\frac{y+10}{4}=\frac{z+17}{14}$

Hint: The given lines have direction ratios proportional, so they are parallel and do not have point of intersection.

14. Find the point of intersection between the lines $\frac{x-1}{2}=\frac{y-3}{2}=\frac{z-2}{-1}$ and $\frac{x-2}{1}=\frac{y-6}{-1}=\frac{z+2}{3}$ Answer: (3, 5, 1)

15. Find the point of intersection between the lines $x-1=\frac{y+2}{3}=\frac{z-4}{-1}$ and $\frac{x}{2}=y-3=\frac{z+3}{4}$

Hint: $x-1=\frac{y+2}{3}=\frac{z-4}{-1}=t$ and $\frac{x}{2}=y-3=\frac{z+3}{4}=s$,

Solving for *t* and *s*, $s=8/5, t=11/5$. And verify the lines do not intersect, as they are skew lines.

16. Find an equation of a plane through the point (-5, 3, 5) which is orthogonal to the line $x=1+3t, y=3-4t, z=1-2t$

17. Name the level curves when a vertical plane passes through the graph of $y^2=x^2+2z^2$

18. Given $r(t)=\ln(20+t^2)\mathrm{i}+\sqrt{6+t^2}\,\mathrm{j}+3e^{7t}\mathrm{k}$, find $r'(t)$

19. Given $r(t)=<e^{2t}\cos t, e^{2t}\sin t, e^{2t}>$, find unit tangent vector at $t=\pi/2$

20. Find the parametric equations for the tangent line to the curve $x=t^5-1, y=t^2+1, z=t^5$ at the point (0, 2, 1)

21. Find the length of the curve $r(t)=<4t, 2\sin t, 2\cos t>, -1\le t\le 3$

22. Find the length of the curve $r(t)=<18t, 9t^2, 9\ln t>,\ t>0$ between the points $(18,9,0)$ and $(72,144,9\ln 4)$

23. Reparametrize the curve $r(t)=<-4-t, -1+2t, -1+3t>$ in terms of arc length *s*.

Hint: $r'(t)=<-1,2,3>$, $|r'(t)|=\sqrt{14}$ and $\frac{ds}{dt}=|r'(t)|$

24. Given acceleration vector $a(t)=2t^2\text{i}+t^{-2}\text{j}+e^t\text{k}$, find the components

$$a_T=\frac{r'(t)\cdot r''(t)}{|r'(t)|},\ a_N=\frac{|r'(t)\times r''(t)|}{|r'(t)|}$$

Hint: $a(t)=2t^2\text{i}+t^{-2}\text{j}+e^t\text{k}=r''(t)$ and $r'(t)=\int\left(2t^2\text{i}+t^{-2}\text{j}+e^t\text{k}\right)dt=\frac{2}{3}t^3\text{i}-t^{-1}\text{j}+e^t\text{k}+\text{c}$,

where $c(t)=c_1\text{i}+c_2\text{j}+c_3\text{k}$ is a vector. And

$$r'(t)=\left(\frac{2}{3}t^3+c_1\right)\text{i}+\left(-t^{-1}+c_2\right)\text{j}+\left(e^t+c_3\right)\text{k}$$

$$=m\text{i}+n\text{j}+p\text{k},\ \text{where } m=\frac{2}{3}t^3+c_1, n=-t^{-1}+c_2, p=e^t+c_3$$

Now

$$a_T=\frac{r'(t)\cdot r''(t)}{|r'(t)|}=\frac{2t^2m+t^{-2}n+e^t p}{\sqrt{m^2+n^2+p^2}}$$

$$a_N=\frac{|r'(t)\times r''(t)|}{|r'(t)|}=\frac{|r'(t)\times r''(t)|}{\sqrt{m^2+n^2+p^2}},\ \textit{where } r'(t)\times r''(t)=\begin{vmatrix}\text{i} & \text{j} & \text{k}\\ m & n & p\\ 2t^2 & t^{-2} & e^t\end{vmatrix}$$

25. Given acceleration vector $a(t)=2t^2\text{i}+t^{-2}\text{j}+e^t\text{k}$, find the components

$$a_T=\frac{r'(t)\cdot r''(t)}{|r'(t)|},\ a_N=\frac{|r'(t)\times r''(t)|}{|r'(t)|}\ \text{when } r'(1)=0$$