SUnMaRC 2008 Abstracts

Invited Speakers:

Edward Burger, Williams College
Zero to Infinity: Great Moments in the Evolution of Numbers
Did ancient cave-people have insights into infinity? How did pre-historic people balance their checkbooks before we had the notion of "number"? Was the Pythagorean Brotherhood really a "brotherhood"? Have you ever prayed to "10"? Does the square-root of 2 really exist? Can you count to 5? Will a randomly selected real number ever equal 1/3? Was Cantor as kooky as his colleagues conjectured? If you've answered "yes" or "no" to any of these questions, then this breathless journey through the history of numbers is for you!

Edward Burger, Williams College
How Always to Win at Limbo
Have you ever gone out with someone for a while and asked yourself: "How close are we?" This presentation will address that question by answering: What does it mean for two things to be close to one another? We'll take a strange look infinite series, dare to mention a calculus student's fantasy, and momentarily consider transcendental meditation. In fact, we'll even attempt to build some very exotic series that can be used if you ever have to flee the country in a hurry: we'll either succeed or fail... you'll have to come to the lecture to find out. Will you be at the edge of your seats? Perhaps; but if not, then you'll probably fall asleep and either way, after the talk, you'll feel refreshed. No matter what, you'll learn a sneaky way to always win at Limbo.

Michelle Zandieh, Arizona State University
Mathematical Reasoning and Proving: Insights from the Theory of Conceptual Blending
The purpose of this report is to describe how several seemingly different phenomena in mathematical reasoning can be seen through one unifying lens with the use of Fauconnier and Turner's theory of conceptual blending. This theory describes how humans reason and learn by combining familiar mental spaces or frames into a new blended space or an integrated network of blended spaces. I use examples of theoretical frameworks from the mathematics education research literature, including data analyzed within these frameworks, and show how they may be reframed using conceptual blending. For this presentation these examples will draw mainly from undergraduate students working to construct a proof that two parallel postulate of Euclidean geometry are equivalent. We will see blending occurring within verbal, symbolic and pictoral reasoning. By explicating these examples I hope to illustrate how conceptual blending may play a role in our understanding of how our students learn and how we may use this knowledge to aid in curriculum design.

Eric Kostelich, Arizona State University
Uncertain Determinism
Simple physical systems that obey Newton's laws can be written as systems of differential equations whose solution, for a given initial condition, is unique. In other words, if one can specify the initial position and velocity of every component, then the state of the system can be predicted, in principle, at every instant in the future. Let us call such a system deterministic. Suppose we want to predict the future state of a

deterministic system where the initial state must be measured somehow. No measurement is perfect, but suppose that, given a positive integer d and enough time and money, it is possible to build an apparatus to measure position and velocity with d decimal digits of accuracy. Furthermore, assume that the ultimate future state of the deterministic system is one of two simple periodic motions. Would it be possible, given these circumstances and unlimited resources, to correctly predict---say 90 percent of the time---which of the two eventual behaviors one will observe given a suitable measurement of the initial condition? I will describe a deterministic system for which the answer is no.

Contributed Talks

1. Brendan Fry, University of Arizona
Insect Population Models
Semelparous periodical insects are those which reproduce only once in their lifetimes, and do so in groups every n years. We will examine a model that is the standard for describing and explaining this behavior in such periodical insects, using two dimensions to represent the juvenile and adult stages of life. In time, we aim to add a third dimension to this model (a predator variable) which will give us a theoretical explanation as to the effect of predators on the unique reproductive life cycles of the insects.

2. Stephanie Williams, Arizona State University
An Ethanol Panacea? The Efficient Transition to Renewable Fuel
America is drastically increasing its investment of resources into the production of biofuels, specifically maize-derived bioethanol. This trend may prove beneficial in the long-term, by reducing reliance on imported oil and decreasing greenhouse gas emissions. However, large government subsidies and vast amounts of agricultural land are being devoted to a pursuit that has not yet been shown to be economically efficient. We explore the market mechanisms that determine how economic agents, technology, production costs, land and prices of petrol and bioethanol affect energy stocks. This, in turn, informs the ultimate decisions that consumers make during a society's transition from reliance on nonrenewable to renewable resources, and whether or not these decisions optimize efficiency.

3. Nicholas Tatonetti, Arizona State University
Software for the Optimization of Kinetic Rate Coefficients though Biochemical Pathways with Metabolic Isotopomers
Data available on the concentrations of metabolic isotopomers in various biochemical pathways, which can be labeled with isotopes have prompted a need for a computational method of isolating the carbon flux throughout these biochemical pathways. In this work, I have developed software for determining the kinetic rate constants in model reaction equations. First, the program generates a set of differential equations based upon a reaction pathway file, which contains a simple syntax for tracking the potentially labeled atoms through a reaction network. Once the set of differential equation has been generated, the program uses a genetic algorithm to optimize the unknown parameters. Genetic algorithms rely on the concepts of natural selection and use the selective pressure of fitness to determine parameters by comparing model simulation results to experimental data. The fitness is used to determine how to allow different sets of parameters to "reproduce" to produce a new generation of parameter sets. When referring to sets of real numbers, "reproduction" means using a linear recombination of different sets to produce "offspring" parameter sets. The program is able to determine

statistically significant sets of parameters after an average of only fifty generations. The software can also be adapted for use as an optimization package for other types of modeling paradigms such as other types of differential equation models or stochastic systems.

4. Aaron Packer, Arizona State University & Angela McBryan, Arizona State University
Ecological Stoichiometry Applied to Algaculture Models
The harvesting of algae for biofuel has been the center of much research over the years. The biggest challenge to successfully harvesting biofuel from algae on an industrial scale is lowering the cost in order to make it profitable. We propose to use the principles of ecological stoichiometry in order to develop a model to maximize the lipid content of a continuously harvested algae population.

5. Micah Mangione, Arizona State University
Issac Newton Never Existed but Paul Erdos Did
In the beginning the Universe was lawless and chaotic and Mathematicians moved upon the face of the unknown. Out of pure will, a few great Greek philosophers said, "Let us make sense of the Cosmos" and mathematics' most fundamental axioms were born. It was in this moment that Euclid decreed, "Let none but the most brilliant of Mathematicians understand the harmonies of this Universe and these few shall carry the torch of Enlightenment." Sound familiar? This is the paraphrased content of many lower level math history courses--the classes which draw the most attendees. Mathematicians and historians alike have a long history of placing credit where none is deserved and forgetting some of the most instrumental minds of Mathematics. This effect results in misnamed equations (and theorems), misguided histories of discoveries and the common misnomer that Mathematicians scurried in the dark until the Great Geniuses of Mathematics lit the Torch of Enlightenment. Who discovered the Gaussian Distribution? (Hint: It wasn't Gauss.)

6. Sean Howe, University of Arizona
Bruhat Intervals in the Symmetric Group
Viewing the symmetric group as a poset under the Bruhat order, one obtains "intervals" by taking the induced directed subgraph of the Hasse diagram between two permutations. We establish an upper bound for the appearance of identity interval classes (identity intervals are intervals that start at the identity permutation), as well as some facts about the self-duality of identity intervals.

7. Christin Bibby, Northern Arizona University
Projective Geometry

8. Jennifer Hale, Arizona State University
Carroll and Vennís Methods of Solving Syllogisms: A History and Contrast
This presentation examines the syllogism methods developed by Venn and Carroll, respectively, as well as noting the positive and negative qualities of each. Later it delves into possible reasons why the Venn method has become so much more widely known than its rival.

9. Russell Latterman, Arizona State University
Designing an Image Classifier for Automobiles

This work shows how to apply machine learning techniques to the automatic design of an automobile detector in digital images. We followed the approach pioneered by Viola and Jones, who obtained a high-performance face detection classifier by applying the statistical technique known as boosting. Our own classifier combines multiple hand-designed car-detection operators, which are used as building blocks in a voting classifier. A novel feature of our approach is the use of color cues in the detection process. We applied AdaBoost to the initial weak classifiers, thereby transforming them into a final strong classifier. A more advanced classifier was later constructed based on our work and has been used by Spirit of Berlin, an autonomous car developed in Germany and at Rice University, during the 2007 Urban Grand Challenge Competition. Since one of the many issues that arise in designing an autonomous vehicle is the problem of detecting other automobiles on the road, developing a good car classifier for video images is highly relevant for the automobile industry.

10. Liz Uribe, University of Arizona & Jeannie Walldren, Northern Arizona University

Solving Systems of Nonlinear Coupled PDE's using Newton's Method

We are seeking solutions u in R^n and v in R^n to the system of coupled superlinear partial difference equations $-Lu+su+u^3+\beta uv^2 = 0$ and $-Lv+sv+v^3+\beta vu^2 = 0$, where L is the matrix corresponding to the Laplacian operator on a graph with Neumann boundary conditions.

11. Thomas Dunn, Northern Arizona University

Critical Points of Complex Polynomials and Rational Functions

Rolle's Theorem tells us where to find critical points for real valued functions; however, it does not apply to complex functions. While there is a version for complex functions, it does not mention the location of critical points. By using special properties of complex polynomials and vector analysis, it can be determined where the critical points can and cannot be.

12. Haokun Xu, University of Arizona

The Constructive Inverse Galois Problem

After learning the basic definitions of Galois theory (which I'll outline) and understanding the statement of the Fundamental Theorem of Galois Theory (which I'll explain), a natural question in Galois theory is to ask "Can we find an example of (insert property here)"? One specialization of this question is the Constructive Inverse Galois Problem over the Rational Numbers, which asks "Given a group G, can we construct a field F such that the Galois group of F over the rational numbers is isomorphic to G?" I'll briefly outline how to construct such fields when G is a commutative group (a well known technique). If time permits (and if I solve this problem before then!), I'll outline how to construct such fields when G is a dihedral group.

13. Ilyssa Summer, Arizona State University

Cancer Stem Cells

The theory of stem cell development within the cell-cell kinetics of normal and cancerous stem cells with a distribution of treatment is investigated with continuous ordinary differential equations. In a normal setting, there are signals that regulate the division of cells; however, cancer cells tend to act "rebelliously" such that these regulations are avoided. The mutated dynamics have the need to be further investigated. The model introduces a cell compartment method with normal and cancer cells differentiating through different stages in the cell cycle including the niche cells in G0, and stem, progenitor and differentiated cells in G1, S, G2 and M with corresponding biological feedback signals to regulate homeostasis within the system. The goal is the find the MTD where a minimum tumor density (cancer cells) will be found as well as minimum drug side effects to the normal cell compartment.

14. Crystal Diaz, Northern Arizona University
A Topological Proof of the Combinatorial Kneser Conjecture
By proving the Kneser conjecture in 1978, László Lovász opened the doors to a new discipline we call topological combinatorics. He showed the math world that topology and discrete mathematics can come together to form beautiful results. This talk involves the discussion of one such result. A famous and likely the first way a combinatorial problem was proven by the application of topological theorems started with Kneser's conjecture. The conjecture is essentially a graph coloring problem, whereas the proof uses the LSB theorem of topology.

15. Andreea Nicolae, Pomona College & Jordan Tirrell, Lafayette College
Nested Chain Partitions of Normalized-Matching Posets
In what ways can we partition a partially ordered set (poset) into linearly ordered subsets (chains)? We will report on recent progress made by our Claremont REU team on a thirty year old conjecture. In particular, two chains C1 and C2 in a finite ranked poset P (a finite poset is ranked if all maximal chains have the same size) are said to be nested if $|C1| < |C2|$ implies that the levels occurring in C1 are a subset of the levels occurring in C2. A thirty-year old conjecture of Griggs gives a sufficient condition, the so-called normalized matching condition, also known as the LYM property for guaranteeing a decomposition of a poset into pairwise nested chains. In this talk, we will present our results in support of the conjecture. As a consequence of our main theorem, the conjecture is true for rank 3 posets of width (size of the largest collection of incomparable elements) less than 12.

16. Jaime Ramos, University of Texas, El Paso
Electrical Network Mixed Problems
This research explores particular inverse problems related to the conductance recoverability in electrical networks. A Mixed Problem is defined when the boundary nodes of the system can be partitioned or divided according with the known data. For instance, I will explain the case where either the voltage or the current is known at all the boundary nodes of the system. To be able to uniquely solve these inverse problems two matrices are introduced: the Response and the Kirchhoff matrices. Using the Schur complement of the Kirchhoff matrix and some of its properties, we are able to recover the conductance of each edge in the system. More general mixed problems, where the data at some nodes is more than just voltage or current, will be presented. Finally, I will introduce a map that recovers the unknown voltage and/or current from the known data and discussed some of its properties. Motivation of the problem: Uniquely determining the voltage on the surface of a patient's heart, based on measurements of voltages and currents on the patient's chest.

17. Jasmin Uribe, University of Arizona
Permutations in Concatenated Zigzag Codes
Coding theory is a branch of mathematics, computer science and electrical engineering that explores the transmission of information across noisy channels. Coding theory is used in data transmission, data storage, and telecommunications. The focus of this project is on concatenated zigzag codes, which are constructed using permutations. We study the effects of permutations on the error-correcting capabilities of the coding

scheme. In conjunction, we explore the behavior of average dispersion in order to further our understanding of randomness of a permutation and find correspondence with error-correction.

18. Amanda Townsend, University of New Mexico
Mathematical Modeling of Nonlinear Optics in Waveguide Arrays
One area of intense research is photonics, where light propagation features are controlled by clever engineering of periodic optical structures. Current research studies the behavior of light beams as they propagate in an array of waveguides. These systems can be modeled by a set of nonlinear ordinary differential equations. In this talk I will discuss the case of a one-dimensional waveguide array. I am still in the early stages of my work, so I will focus on the problem and its setup.

19. Tatiana Moyer, Arizona State University
Pebbling Balloon Animals
A configuration C of pebbles distributed on the vertices of an undirected graph G is solvable if any vertex of G can be pebbled in a finite number of pebbling moves. A pebbling move from adjacent vertices u to v is a new configuration in which two pebbles are removed from u and one pebble is added to v. A vertex v is "pebbled" if after a finite number of pebbling moves, the number of pebbles on v is at least one. Define the pebbling number $\pi(G)$ as the minimum size of C such that C is solvable. In this paper, we give tight upper and lower bounds for the pebbling numbers of graphs of connectivity one, each of whose blocks have diameter at most two. This work is the beginning of an attempt to study the pebbling numbers of graphs in terms of their block structure.

20. Taylor Hines, Arizona State University
On Near-UCycles for Subsets
A Universal Cycle (UCycle) for k-subsets of an n-set is a cyclic sequence containing every such subset exactly once as a contiguous sequence. Invented by Chung, Diaconis, and Graham in 1992, few examples are known to exist. A Ucycle packing (covering) is a sequence in which each k-subset appears at most (at least) once. A near-UCycle packing (covering) is a packing that misses (repeats) asymptotically few subsets. In this paper we prove the existence of near-Ucycle packings for all k.

21. Alan Hogan, Arizona State University
Efficient Means of Finding Hamiltonian Circuits
Given an undirected graph, we construct a Hamiltonian circuit (in polynomial time). Here we use a potentially more usable and efficient algorithm, albeit only able to solve the problem for graphs for which every vertex has degree at least n/2, where n is the number of vertices in the graph. That is, for a graph G of size n, consisting of vertices A = $\{a_1, a_2, \ldots, a_{n-1}, a_n\}$, and d(a) is the degree of vertex $a \in A$, then our solution applies when for all A, $d(a) \geq n/2$ is true.