

**M E M O R A N D U M**

DATE: 04/06/2023

TO: Faculty and Students

FROM: Professor(s) Yang Kuang & Eric Kostelich  
Chair/Co-Chairs of Duane Harris  
Defense for the PhD in Applied Mathematics  
Committee Members Mark Preul  
Sharon Crook  
Carl Gardner

**DEFENSE ANNOUNCEMENT**

Candidate: Duane Harris

Defense Date: Friday, April 7, 2023

Defense Time: 12:00 PM

Virtual Meeting Link: <https://asu.zoom.us/j/81127309679>

Title: Modeling Brain Cancer Progression using Reaction-Diffusion Equations with Minimal Parameters

Please share this information with colleagues and other students, especially those studying in similar fields. Faculty and students are encouraged to attend. The defending candidate will give a 40 minute talk, after which the committee members will ask questions. There may be time for questions from those in attendance. But, guests are primarily invited to attend as observers and will be excused when the committee begins its deliberations or if the committee wishes to question the candidate privately.

**ABSTRACT**

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## ABSTRACT

We present our numerical and analytical work pertaining to models that describe the growth and progression of glioblastoma multiforme (GBM), an aggressive form of primary brain cancer. Two reaction-diffusion models are used: the Fisher-Kolmogorov-Petrovsky-Piskunov (FKPP) equation and a 2-population model that divides the tumor into actively proliferating and quiescent (or necrotic) cells.

The numerical portion of this talk focuses on simulating GBM expansion in patients undergoing treatment for recurrence of tumor following initial surgery and chemoradiation. The models are simulated on 3-dimensional brain geometries derived from magnetic resonance imaging (MRI) scans provided by the Barrow Neurological Institute. The study consists of 17 clinical time intervals across 10 patients that have been followed in detail, each of whom shows significant progression of tumor over a period of 1 to 3 months on sequential follow up scans. A Taguchi sampling design is implemented to estimate the variability of the predicted tumors by using 144 different choices of model parameters. In 9 cases, model parameters can be identified such that the simulated tumor contains at least 40 percent of the volume of the observed tumor. Finally, we discuss some potential improvements that can be made to the simulations by utilizing perfusion MRI data to inform the initialization of the model.

In the analytical portion of the talk, we begin by describing a positively invariant region for our 2-population model, which guarantees that solutions remain positive and bounded from above for all time. Then, we present the results of our rigorous steady-state analysis in which we derived the critical patch size associated with our model. The critical patch (KISS) size is the minimum habitat size needed for a population to survive in a region. Habitats larger than the critical patch size allow a population to persist, while smaller habitats lead to extinction. We determine that the critical patch size of our model is consistent with that of the FKPP equation (one of the first reaction-diffusion models proposed for GBM) and does not depend on parameters pertaining to the quiescent/necrotic population. The critical patch size may indicate that GBM tumors have a minimum size depending on the location in the brain. We also highlight a theoretical relationship between the size of a GBM tumor at steady-state and its maximum cell density, which has potential applications for patient-specific parameter estimation based on magnetic resonance imaging data.