Course: MAT 425
Numerical Analysis Final Exam and
Mathematics Department Qualifier Exam
Spring 2006

Student Name: _____

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Circle the type of exam: **Final** **Qualifier** **Both**

Circle name of instructor: **Gelb/Renaut** **Lopez**

Please write your name on the top of each page of your answer
Only use one side of each sheet of paper.
No books or handouts allowed

1. (25 points +5 bonus if you do the calculations) Consider the following table of approximate integrals I_n produced using Simpson's rule.

n	I_n
2	.28451779686
4	.28559254576
8	.28570247748
16	.28571317731
32	.28571418363
64	.28571427643

How would you

(a) Predict the order of convergence of I_n to I? That is, if $I - I_n \approx c/n^p$, then what is p?

(b) Does this appear to be valid form for the error for these data?

(c) Predict a value of c

(d) find the error in I_{64}.

(e) Find how large n should be if I_n is to be error by less than 10^{-11}?

2. Consider the initial value problem

$$y' = y^{\frac{1}{3}}, \quad y(0) = 0.$$

(a) Find the analytic solution.

(b) Using stepsize $h = 0.1$ and Euler's method calculate the numerical solution on the interval $[0, 1]$.

(c) Explain why the numerical solution fails to approximate the exact solution.

3. Consider the boundary value problem for $x \in [0, 1]$:

$$y'' = 4(y - x),$$

with $y(0) = 0$ and $y(1) = 2$.

(a) Show that the BVP has a unique solution.

(b) Describe how the linear shooting method is used to solve the BVP.

4. Consider the following scheme for the one-way wave equation $u_t = au_x$ with periodic boundary conditions and $u(x,0) = f(x)$:

$$u_j^{n+1} = u_j^{n-1} + \frac{a\Delta t}{\Delta x}(\frac{4}{3}(u_{j+1}^n - u_{j-1}^n) - \frac{1}{6}(u_{j+2}^n - u_{j-2}^n)).$$

   (a) Draw a sketch of the stencil for this scheme.

   (b) What is the order of accuracy for this scheme?

   (c) Use Von Neumann analysis to determine its stability condition.

   (d) Is this scheme dissipative?