

Internet Mat117 Formulas and Concepts

1. The **distance** between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the plane is

$$d(A, B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

2. The **midpoint** of the line segment from $A(x_1, y_1)$ to $B(x_2, y_2)$ is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

3. The **standard form** for the equation of the circle with center (h, k) and radius r :

$$(x - h)^2 + (y - k)^2 = r^2.$$

4. The **slope** m of a non-vertical line that passes through the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

5. The **point-slope form** of the equation of the line that passes through the point $P_1(x_1, y_1)$ and has a slope m :

$$y - y_1 = m(x - x_1).$$

6. The **slope-intercept form** of the equation of the line that has a slope m and y -intercept b :

$$y = mx + b.$$

7. The equation of the **vertical line** though the point $P(a, b)$:

$$x = a.$$

8. The equation of the **horizontal line** though the point $P(a, b)$:

$$y = b.$$

9. The **general** equation a line:

$$Ax + By + C = 0 \quad A \text{ and } B \text{ not both zero.}$$

10. Two non-vertical lines with slopes m_1 and m_2 are parallel if and only if $m_1 = m_2$.
11. Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_2 = -\frac{1}{m_1}$.
12. $y = f(x) + c$, $c > 0$. Shift the graph of $y = f(x)$ upward c units.
13. $y = f(x) - c$, $c > 0$. Shift the graph of $y = f(x)$ downward c units.
14. $y = f(x + c)$, $c > 0$. Shift the graph of $y = f(x)$ left c units.
15. $y = f(x - c)$, $c > 0$. Shift the graph of $y = f(x)$ right c units.
16. $y = -f(x)$. Reflect the graph of $y = f(x)$ with respect to the x -axis.
17. $y = f(-x)$. Reflect the graph of $y = f(x)$ with respect to the y -axis.
18. $y = af(x)$, $a > 1$. Stretch the graph of $y = f(x)$ vertically by a factor of a .
19. $y = af(x)$, $0 < a < 1$. Compress the graph of $y = f(x)$ vertically by a factor of a .
20. The **standard form** for the quadratic function $f(x) = ax^2 + bx + c$:

$$f(x) = a(x - h)^2 + k$$

The standard form can be obtained by the procedure of completing the square.

The graph of $f(x)$ is a parabola with **vertex** (h, k) .

The parabola is open upward if $a > 0$ and open downward if $a < 0$.

21. **Vertex formula** of a quadratic function $y = f(x) = ax^2 + bx + c$:

$$x = -b/2a, \quad y = f(-b/2a).$$

If $a > 0$, then the **minimum value** of f is $f(-b/2a)$ when $x = -b/2a$.

If $a < 0$, then the **maximum value** of f is $f(-b/2a)$ when $x = -b/2a$.

22. **The quadratic formula.** The solutions for the quadratic equation $ax^2 + bx + c = 0$:

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

23. **Algebra of functions.** Let f and g be two functions with domain A and B . Then

$$(f + g)(x) = f(x) + g(x) \text{ with domain } A \cap B.$$

$$(f - g)(x) = f(x) - g(x) \text{ with domain } A \cap B.$$

$$(fg)(x) = f(x)g(x) \text{ with domain } A \cap B.$$

$$(f/g)(x) = f(x)/g(x) \text{ with domain } \{x \in A \cap B | g(x) \neq 0\}.$$

24. **Composition function** Given two functions f and g . The composition function of f and g is denoted by $(f \circ g)(x)$ and defined by

$$(f \circ g)(x) = f(g(x)).$$

25. **Inverse Function.** Let f be a one-to one function with domain A and range B . The the inverse function of $f(x)$ is denoted by f^{-1} is defined by

$$f^{-1}(y) = x \text{ if and only if } f(x) = y.$$

The domain of f^{-1} is B .

The range of f^{-1} is A .

26. **How to find the inverse function.**

Write $y = f(x)$.

Solve the equation for x in terms of y .

Interchange the variables x and y . The resulting equation is $y = f^{-1}(x)$.

27. The graph of f^{-1} is obtained by reflecting the graph of f with respect to the line $y = x$.

28. **Polynomial function.** Important terms and theorems:

Degree of a polynomial function $P(x)$.

Coefficients of a polynomial function $P(x)$.

Constant term or constant coefficient of a polynomial function $P(x)$.

Leading coefficient of a polynomial function $P(x)$.

Leading term of polynomial function $P(x)$.

Zero of a polynomial function $P(x)$. $P(c) = 0$ if and only if $x - c$ is a factor of $P(x)$.

Multiplicity of a zero of a polynomial function $P(x)$.

The end behavior of a polynomial function $P(x)$. Find where $P(x)$ approaches when $x \rightarrow \infty$ or $x \rightarrow -\infty$.

Rational zeros Theorem.

Long Division.

Synthetic Division.

Factoring a polynomial function $P(x)$.

Graphing a polynomial function $P(x)$.

29. **Rational function.** Important terms and theorems:

A rational function has the form of $R(x) = \frac{P(x)}{Q(x)}$ where the $P(x)$ and $Q(x)$ are polynomial functions.

The x -intercepts of a rational function: where the numerator is zero ($P(x) = 0$) assuming that $P(x)$ and $Q(x)$ have no common factors.

The y -intercept of a rational function: substitute $x = 0$.

The **vertical asymptotes** of a rational function: where the denominator is zero ($Q(x) = 0$) assuming that $P(x)$ and $Q(x)$ have no common factors. In this case the rational function is unbounded at the vertical asymptotes.

The **horizontal asymptote** of a function: the line $y = b$ is a horizontal asymptote of the function $y = f(x)$ if $y \rightarrow b$ as $x \rightarrow \infty$ or $x \rightarrow -\infty$.

How to find the horizontal asymptote of a rational function $R(x) = \frac{P(x)}{Q(x)}$.

Case1: if the degree of $P(x) <$ the degree of $Q(x)$ then the horizontal asymptote $y = 0$.

Case2: if the degree of $P(x) =$ the degree of $Q(x)$ then the horizontal asymptote $y =$ ratio of the leading coefficients.

Case3: if the degree of $P(x) >$ the degree of $Q(x)$ then no horizontal asymptote.

Graphing a rational function with your graphing calculator.

30. **Complex numbers.** Important terms and theorems:

Arithmetic of complex numbers: addition, subtraction, multiplication and division.

Complex conjugate of a complex number $a + bi$ is $a - bi$.

Solving quadratic equation with complex roots. The complex roots of a quadratic equations are complex conjugates.

Polynomial function of degree n has n complex roots.

31. For $a > 0$ **exponential function with base a** is defined by

$$f(x) = a^x.$$

For $a \neq 1$, the domain f is $(-\infty, \infty)$ and the range of f is $(0, \infty)$.

If $a > 1$ then $f(x) = a^x$ is strictly increasing and if $a < 1$ then $f(x) = a^x$ is strictly decreasing.

32. The **natural exponential function** is the exponential function

$$f(x) = e^x$$

with base e .

33. **Compound interest** is calculated by the formula

$$A(t) = P(1 + r/n)^{nt} \text{ where}$$

$A(t)$ = the amount after t years

P =principal

r =interest rate

n =number of times interest is compounded per year

t =number of years.

34. **Continuously compounded interest** is calculated by the formula

$$A(t) = Pe^{rt} \text{ where}$$

$A(t)$ = the amount after t years.

P =principal

r =interest rate

t =number of years.

35. A population that experiences **exponential growth** increases according to the model

$$N(t) = N_0e^{rt} \text{ where}$$

$N(t)$ = the population at time t

N_0 =initial size of the population

r =relative rate of growth

t =time

36. **Radio active decay** of a radio active material can be modelled by the formula

$$Q(t) = Q_0 e^{-kt} \text{ where}$$

$Q(t)$ = the amount of material at time t

Q_0 = initial amount of material

$k > 0$ constant, depends on the material

t = time

37. Let a be a positive number with $a \neq 1$. The **logarithmic function with base a** , denoted by \log_a , is defined by

$$\log_a x = y \text{ if and only if } a^y = x$$

So, $\log_a x$ is the exponent to which the base a must be raised to get x .

38. The logarithm with base e is called the **natural logarithm** and it is denoted by \ln .

$$\ln x = \log_e x$$

and

$$\ln x = y \text{ if and only if } e^y = x.$$

39. The logarithm with base 10 is denoted by \log .

$$\log x = \log_{10} x$$

and

$$\log x = y \text{ if and only if } 10^y = x.$$

40. **Properties of logarithm**

1. $\log_a 1 = 0$ because we have to raise a to the power 0 to get 1.
2. $\log_a a = 1$ because we have to raise a to the power 1 to get a .
3. $\log_a a^y = y$ because we have to raise a to the power y to get a^y .
4. $a^{\log_a x} = x$ because $\log_a x$ is the power to which a has to be raised to get x .

41. Properties of natural logarithm

1. $\ln 1 = 0$ because we have to raise e to the power 0 to get 1.
2. $\ln e = 1$ because we have to raise e to the power 1 to get e .
3. $\ln e^y = y$ because we have to raise e to the power y to get e^y .
4. $e^{\ln x} = x$ because $\ln x$ is the power to which e has to be raised to get x .

42. Properties of logarithm with base 10

1. $\log 1 = 0$ because we have to raise 10 to the power 0 to get 1.
2. $\log 10 = 1$ because we have to raise 10 to the power 1 to get 10.
3. $\log 10^y = y$ because we have to raise 10 to the power y to get 10^y .
4. $10^{\log x} = x$ because $\log x$ is the power to which 10 has to be raised to get x .

43. Laws of logarithms

Let $a > 0, a \neq 1$. Let $A > 0, B > 0$ and C be any real number.

1. $\log_a(AB) = \log_a A + \log_a B$
2. $\log_a(A/B) = \log_a A - \log_a B$
3. $\log_a(A^C) = C \log_a A$.

44. Laws of natural logarithms

Let $A > 0, B > 0$ and C be any real number.

1. $\ln(AB) = \ln A + \ln B$
2. $\ln(A/B) = \ln A - \ln B$
3. $\ln(A^C) = C \ln A$

45. Laws of logarithms with base 10

Let $A > 0, B > 0$ and C be any real number.

1. $\log(AB) = \log A + \log B$
2. $\log(A/B) = \log A - \log B$
3. $\log(A^C) = C \log A$

46. Change of base formula

Let $a > 0, a \neq 1, b > 0, b \neq 1$ and $x > 0$.

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Consequently,

$$\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}$$

47. A **sequence** is a function f whose domain is the set of natural numbers. The values $f(1), f(2), f(3) \dots$ are called the **terms** of the sequence.
48. For the sequence

$$a_1, a_2, a_3, \dots, a_n, \dots$$

the **partial sums** are

$$S_1 = a_1$$

$$S_2 = a_1 + a_2$$

$$S_3 = a_1 + a_2 + a_3$$

$$S_4 = a_1 + a_2 + a_3 + a_4$$

$$S_n = a_1 + a_2 + a_3 + a_4 + \dots + a_n.$$

S_1 is called the **first partial sum**, S_2 is called the **second partial sum** and so on, S_n is called the **n -th partial sum**. The sequence $S_1, S_2 \dots S_n \dots$ is called the **sequence of partial sums**.

49. Given a sequence

$$a_1, a_2, a_3, \dots, a_n, \dots$$

we can write the sum of the first n terms using **summation notation** or **sigma notation** as follows:

$$\sum_{k=1}^n a_k = a_1 + a_2 + a_3 + a_4 + \dots + a_n.$$

50. An **arithmetic sequence** is a sequence of the form

$$a_1, a_1 + d, a_1 + 2d, a_1 + 3d, a_1 + 4d, \dots$$

The number a_1 is the **first term**, and d is the **common difference** of the sequence. The **n -th term** of an arithmetic sequence is given by

$$a_n = a + (n - 1)d.$$

51. For the arithmetic sequence $a_n = a_1 + (n - 1)d$, the **n -th partial sum**

$$S_n = a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) + \dots + [a_1 + (n - 1)d]$$

is given by either of the following formulas

$$1. S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

$$2. S_n = n\left(\frac{a_1 + a_n}{2}\right).$$

52. An **geometric sequence** is a sequence of the form

$$a_1, a_1r, a_1r^2, a_1r^3, a_1r^4, \dots$$

The number a_1 is the **first term**, and r is the **common ratio** of the sequence. The **n -th term** of an arithmetic sequence is given by

$$a_n = a_1r^{n-1}.$$

53. For the geometric sequence $a_n = a_1r^{n-1}$, the **n -th partial sum**

$$S_n = a_1 + (a_1r) + (a_1r^2) + (a_1r^3) + \dots + [a_1r^{n-1}]$$

is given by the following formula

$$S_n = a_1 \frac{1 - r^n}{1 - r}.$$

54. If $r < 1$, then the **infinite geometric series**

$$a_1 + (a_1r) + (a_1r^2) + (a_1r^3) + \dots + [a_1r^{n-1} + \dots$$

has the sum

$$S = \frac{a_1}{1 - r}.$$