INTERNET MAT 117

Review Problems

(1) Let us consider the circle with equation

$$x^2 + y^2 + 2x + 3y + \frac{3}{4} = 0.$$

(a) Find the standard form of the equation of the circle given above.

(b) Find the center and the radius of the circle given above.

(2) (a) Find the slope intercept form of the line passing through $(-2, 5)$ and parallel to the line $5x - 3y = 8$.

(b) Find the slope intercept form of the line passing through $(-2, 5)$ and perpendicular to the line $5x - 3y = 8$.

(3) Find the domain of the following functions. Give your answer in interval notation.

(a) $g(x) = \dfrac{\sqrt{x-1}}{5-x}$

(b) (2.5 points) $f(x) = \dfrac{x+1}{4-x^2}$

(4) A small business buys a computer for \$ 4,000. After 4 years the value of the computer is expected to be \$ 200. For accounting purposes, the business uses linear depreciation to assess the value of the computer at a given time. This means the if V is the value of the computer at the time t, then a linear equation is used to relate V and t.

(a) Find a linear equation that relates V and t.

(b) Find the depreciated values of the computer 3 years from the date of purchase.

(5) Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+2}$

(a) Find $(\frac{f}{g})(x)$ and give the domain of $\frac{f}{g}$.

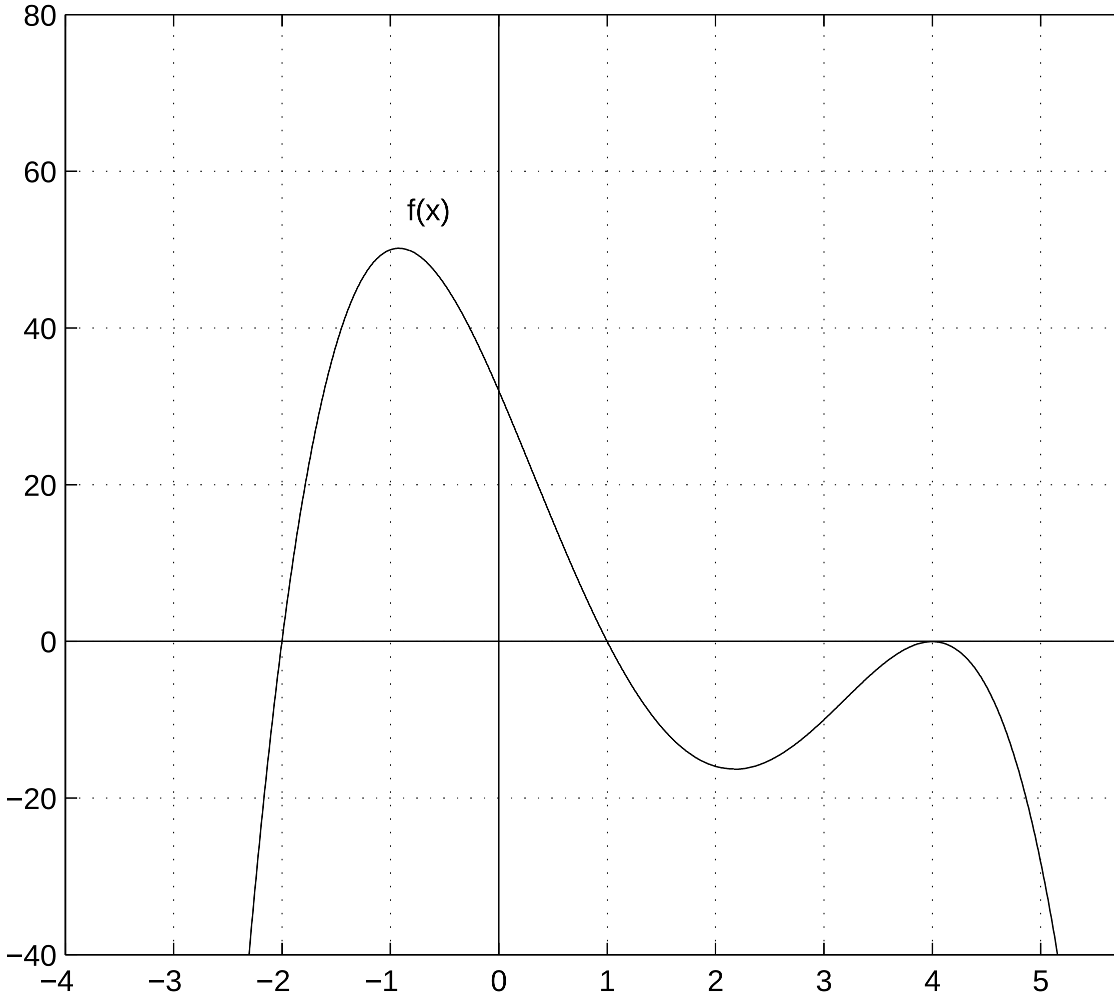
(b) Find $(f \circ g)(x)$ and $(g \circ f)(x)$

(6) Let $f(x) = -2x + 4$.

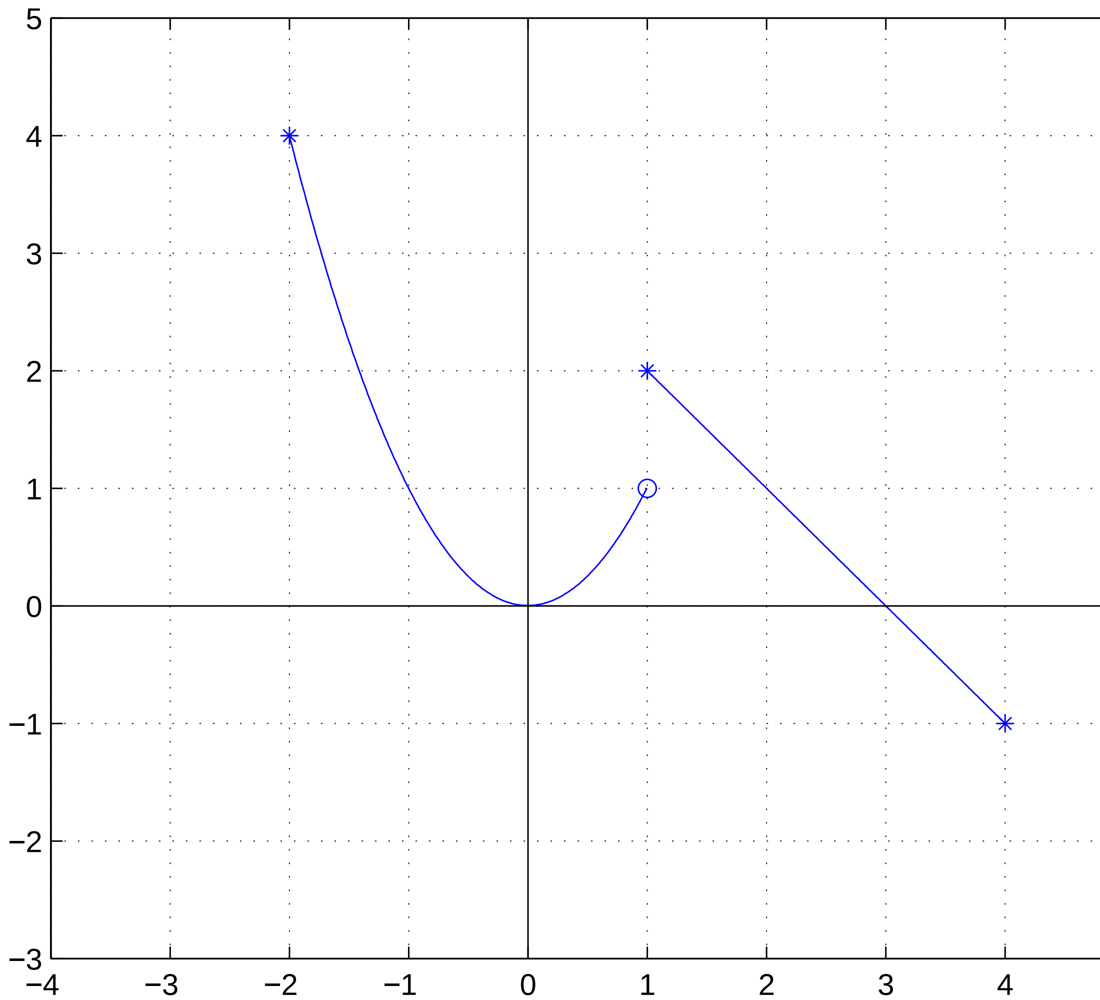
(a) Find $f^{-1}(x)$.

(b) Graph f and f^{-1} on the same coordinate axis.

(7) Use the graph of $f(x)$ below to sketch the graph of $g(x)$ it if is known that $g(x) = f(x-1) + 20$. Draw $g(x)$ on the same set of axes as the given graph.



(8) Consider the graph of the function $f(x)$ below. Answer the following questions:



(a) Use interval notation to state the domain of $f(x)$.
(b) Use interval notation to state the range of $f(x)$.
(c) State the interval where the function $f(x)$ increases.
(d) State the interval where the function $f(x)$ decreases.
(e) State the values of $f(1)$ and $f(4)$.

(9) Let $P(x) = x^3 - 3x^2 - 9x - 5$

(a) Determine all the possible rational zeros for $P(x)$.

(b) Factor the polynomial $P(x)$ completely using the procedure of long division or synthetic division.

(c) Find all the zeros of $P(x)$ and state the multiplicity for each zero. Give the exact answer. No decimals.

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(d) Find where $P(x)$ approaches when $x \to \infty$ and when $x \to -\infty$.

(e) (1 points) Sketch the graph of $P(x)$ indicating the zeros.

(10) Consider the polynomial $P(x) = x(x+2)^2(x-3)^3$.

(a) Find all zeros of $P(x)$ and state their multiplicities.

(b) Where $P(x)$ approaches when $x \to \infty$ and when $x \to -\infty$.

(c) Sketch the graph of the function $P(x)$ indicating the x-intercepts (the zeros).

(11) Evaluate the expression $\dfrac{1-3i}{2+5i}$ and write the result in the $a+bi$ form .

(12) Find all the real zeros of the polynomial $x^3+2x^2-10x-15$. Use the quadratic formula if it is necessary. Use radicals, don't use decimals.(Hint: First find the rational zeros.)

(13) Find all the complex zeros of x^2+2x+2. Use radicals don't use decimals.

(14) Find the quotient $(Q(x)$ and the remainder $(R(x)$ using the long division.

$$\frac{3x^4 - 5x^3 + 4x + 3}{x^2 + x + 3}$$

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(15) (5 points) An object is projected upward from the top of a building. The height of the object in meters is described by the function $h(t) = -4.9t^2 + 40t + 10$, where t is in seconds and corresponds to the moment the object is projected.

(a) Determine the height of the building.

(b) Algebraically determine for what value of t the object reaches the maximum height and determine this maximum height.(Hint use the vertex formula.) Check your answer with your calculator.

(c) Algebraically determine when the object reaches the ground. (Hint use the quadratic formula). Check your answer with your calculator.

(16) The function $p(x) = -x^2 + 46x - 360$ models the daily profit in hundreds of dollars for a company that manufactures x computers daily. (You may show your work algebraically or graphically, which includes a sketch of the graph.)

(a) How many computers should be manufactured each day to maximize profit?

(b) What is the maximum daily profit?

(17) (a) Consider the rational function $R(x) = \dfrac{x^2 - x - 6}{2x^2 - 2}$.

(i) Find the vertical asymptotes of $R(x)$ if there is any.

(ii) Find the horizontal asymptote of $R(x)$ if there is any.

(iii) Find the x-intercepts of $R(x)$ if there is any.

(iv) Find the y-intercept of $R(x)$ if there is any.

(b) Consider the rational function $R(x) = \dfrac{x^2 - x - 6}{2x^2 - 2}$.

(i) Find the vertical asymptotes of $R(x)$ if there is any.

(ii) Find the horizontal asymptote of $R(x)$ if there is any.

(iii) Find the x-intercepts of $R(x)$ if there is any.

(iv) Find the y-intercept of $R(x)$ if there is any.

(c) Consider the rational function $R(x) = \dfrac{x^2 - x - 6}{2x^2 - 2}$.

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(ii) Find the horizontal asymptote of $R(x)$ if there is any.

(iii) Find the x-intercepts of $R(x)$ if there is any.

(iv) Find the y-intercept of $R(x)$ if there is any.

(18) The number of deer in a state forest can be modeled using the the rational function

$$N(t) = 4500(\frac{6t^2 + 1}{3t^2 + 1})$$

where t is the time in years after the herd was first introduced.

(a) Approximately how many deer are there 5 years after they were introduced into the forest?

(b) In the long run, how many deer will there be in this state forest? Explain the reasoning behind your answer. Also explain what possible real-world factors might limit the number of deer in the forest.

(19) Solve for x.

(a) $\ln x + \ln(x - 15) = \ln 34$.

(b) $\log x + \log(x + 3) = 1$.

(c) $e^{2x} - 3e^{x} - 10 = 0$.

(20) (a) Use logarithms to find the solution correct to 3 decimal places.

$$e^{2x-3} = e^{2x+1}$$

(b) Use the Laws of logarithms to rewrite the following expression in a form with no logarithm of a product, quotient or power.

$$x^4\sqrt{\frac{y^3}{z^2}}$$

(c) Rewrite the following expression as a single logarithm.

$$2\log(x+1) - 3\log(y-5) + \log z$$

(21) Find the time required for an investment of 2500 dollars to grow to 9000 dollars at an interest rate of 6.5 percent per year, compounded quarterly.

(22) You are in a group of city planners that is trying to determine whether or not to expand your water supply facilities. To aid in your decision, you will use the information from the last two census figures for the city, showing a population of 80,000 at the start of 1990 and a population of 88,300 at the start of 2000. Knowing that you can currently supply enough water for 95,000 people use the exponential model $Q(t) = Q_0 \cdot e^{kt}$ to determine during what year you will no longer have enough water to meet the needs of your city.

(23) Find the half-life of a radioactive substance if 200 grams of the substance decays to 180 grams in 2 year.

(24) (a) Find the exponential function $f(x) = a^x$ whose graph goes through the point. $(2, 10.3)$.

(b) For the logarithmic function $\log_3(x-4) = y$ find the domain and the x-intercept. Sketch the graph of the function.

(25) Find all the solutions of the following systen of equations.

(a)

$$8x - 3y = -3$$

$$5x - 2y = -1$$

(b)

$$x - 2y = 2$$

$$y^2 - x^2 = 2x + 4$$

(c)

$$x + 4y = 8$$

$$3x + 12y = 2$$

(d)

$$3x^2 + 4y = 17$$

$$2x^2 + 5y = 2$$

(26) The 12th term of an arithmetic sequence is 32 and the 5th term is 18. Find the difference d and the 20th term.

(27) An architect designs a theater with 15 seats in the first row, and 18 in the second , 21 in the third , and so on. If the theater is to have a seating capacity of 870 , how many rows must the architect use in his design.

(28) The common ratio in a geometric sequence is 2/5 and the 4th term is 5/2. Find the 3rd term.

(29) Find the sum.

$$1 + 3 + 9 + 27 + \ldots + 2187$$

(30) Determine whether the following sequence arithmetic or geometric and find the 8th term of the sequence.

$$\frac{27}{8}, \frac{-9}{4}, \frac{3}{2}, -1$$